CP and T violation in (long) long baseline neutrino oscillation experiments

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Abstract

We consider possibilities of observing CP-violation effects in neutrino oscillation experiments with low energy ($\sim$ several hundreds MeV).

1 Introduction

Many experiments and observations have shown evidences for neutrino oscillation one after another. The solar neutrino deficit has long been observed(1; 2; 3; 4; 5). The atmospheric neutrino anomaly has been found(6; 7; 8; 9) and recently almost confirmed by SuperKamiokande(10). There is also another suggestion given by LSND(11). All of them can be understood by neutrino oscillation and hence indicates that neutrinos are massive and there is a mixing in lepton sector(12).

Since there is a mixing in lepton sector, it is quite natural to imagine that there occurs CP violation in lepton sector. Several physicists have considered whether we may see CP-violation effect in lepton sector through long baseline neutrino oscillation experiments. First it has been studied in the context of currently planed experiments(13; 14; 15; 16; 17) and recently in the context of neutrino factory(18; 19; 20; 21).

The use of neutrinos from muon beam has great advantages compared with those from pion beam(22). Neutrinos from $\mu^+(\mu^-)$ beam consist of pure ν_e and $\bar{\nu}_\mu$ ($\bar{\nu}_e$ and ν_μ) and will contain no contamination of other kinds of neutrinos. Also their energy distribution will be determined very well.

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In this proceedings, we will consider how large CP-violation effect we will see in oscillation experiments with low energy neutrino from muon beam. Such neutrinos with high intensity will be available in near future(23). We will consider three active neutrinos without any sterile one by attributing the solar neutrino deficit and atmospheric neutrino anomaly to the neutrino oscillation.

2 CP violation in long baseline neutrino oscillation experiments

Here we consider neutrino oscillation experiments with baseline $L \sim$ several hundreds km.

2.1 Oscillation probability and its approximated formula

First we derive approximated formulas(15) of neutrino oscillation to clarify our notation.

We assume three generations of neutrinos which have mass eigenvalues $m_i (i = 1, 2, 3)$ and MNS mixing matrix $U(24)$ relating the flavor eigenstates $\nu_\alpha (\alpha = e, \mu, \tau)$ and the mass eigenstates in the vacuum $\nu'_i (i = 1, 2, 3)$ as

$$\nu_\alpha = U_{\alpha i} \nu'_i. \tag{1}$$

We parameterize $U(25; 26; 27)$ as

$$
\begin{aligned}
U &= e^{i\psi\lambda_7} \Gamma e^{i\phi\lambda_5} e^{i\omega\lambda_2} \\
&= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\psi & s_\psi \\ 0 & -s_\psi & c_\psi \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} c_\phi & 0 & s_\phi \\ 0 & 1 & 0 \\ -s_\phi & 0 & c_\phi \end{pmatrix} \begin{pmatrix} c_\omega & s_\omega & 0 \\ -s_\omega & c_\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} c_\phi c_\omega & c_\phi s_\omega & s_\phi \\ -c_\psi s_\omega - s_\psi s_\phi c_\omega e^{i\delta} & c_\psi c_\omega - s_\psi s_\phi s_\omega e^{i\delta} & s_\psi c_\phi e^{i\delta} \\ s_\psi s_\omega - c_\psi s_\phi c_\omega e^{i\delta} & -s_\psi c_\omega - c_\psi s_\phi s_\omega e^{i\delta} & c_\psi c_\phi e^{i\delta} \end{pmatrix},
\end{aligned}
\tag{2}
$$

where $c_\psi = \cos\psi$, $s_\phi = \sin\phi$, etc.

The evolution equation of neutrino with energy E in matter is expressed as

$$i\frac{d\nu}{dx} = H\nu, \tag{3}$$

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where

$$H \equiv \frac{1}{2E}\tilde{U}\text{diag}(\tilde{m}_1^2, \tilde{m}_2^2, \tilde{m}_3^2)\tilde{U}^\dagger, \tag{4}$$

with a unitary mixing matrix $\tilde{U}$ and the effective mass squared $\tilde{m}_i^2$'s ($i = 1, 2, 3$). The matrix $\tilde{U}$ and the masses $\tilde{m}_i$'s are determined by(28; 29; 30)

$$\tilde{U}\begin{pmatrix} \tilde{m}_1^2 & & \\ & \tilde{m}_2^2 & \\ & & \tilde{m}_3^2 \end{pmatrix}\tilde{U}^\dagger = U\begin{pmatrix} 0 & & \\ & \delta m_{21}^2 & \\ & & \delta m_{31}^2 \end{pmatrix}U^\dagger + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix}. \tag{5}$$

Here $\delta m_{ij}^2 = m_i^2 - m_j^2$ and

$$a \equiv 2\sqrt{2}G_\text{F}n_\text{e}E = 7.56 \times 10^{-5}\text{eV}^2 \cdot \left(\frac{\rho}{\text{g cm}^{-3}}\right)\left(\frac{E}{\text{GeV}}\right), \tag{6}$$

with the electron density, n_e and the averaged matter density(31), ρ. The solution of eq.(3) is then

$$\nu(x) = S(x)\nu(0) \tag{7}$$
$$S \equiv \text{T}\,\text{e}^{-\text{i}\int_0^x \text{d}sH(s)} \tag{8}$$

(T being the symbol for time ordering), giving the oscillation probability for $\nu_\alpha \to \nu_\beta(\alpha, \beta = \text{e}, \mu, \tau)$ at distance L as

$$P(\nu_\alpha \to \nu_\beta; E, L) = |S_{\beta\alpha}(L)|^2. \tag{9}$$

Note that $P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$ is related to $P(\nu_\alpha \to \nu_\beta)$ through $a \to -a$ and $U \to U^*$(i.e. $\delta \to -\delta$). Similarly, we obtain $P(\nu_\beta \to \nu_\alpha)$ from eq.(9) by replacing $\delta \to -\delta$, $P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$ by $a \to -a$.

Attributing both solar neutrino deficit and atmospheric neutrino anomaly to neutrino oscillation, we can assume $a, \delta m_{21}^2 \ll \delta m_{31}^2$. The oscillation probabilities in this case can be considered by perturbation(15). With the additional conditions

$$\frac{aL}{2E} = 1.93 \times 10^{-4} \cdot \left(\frac{\rho}{\text{g cm}^{-3}}\right)\left(\frac{L}{\text{km}}\right) \ll 1 \tag{10}$$

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and

$$\frac{\delta m_{21}^2 L}{2E} = 2.53 \frac{(\delta m_{21}^2 / \text{eV}^2)(L/\text{km})}{E/\text{GeV}} \ll 1, \tag{11}$$

the matrix S (8) is given by

$$S(x) \simeq e^{-iH_0 x} + e^{-iH_0 x}(-i) \int_0^x ds\, H_1(s), \tag{12}$$

where

$$H_0 = \frac{1}{2E} U \begin{pmatrix} 0 & & \\ & 0 & \\ & & \delta m_{31}^2 \end{pmatrix} U^\dagger \tag{13}$$

$$H_1(x) = e^{iH_0 x} H_1 e^{-iH_0 x}, \tag{14}$$

$$H_1 = \frac{1}{2E} \left\{ U \begin{pmatrix} 0 & & \\ & \delta m_{21}^2 & \\ & & 0 \end{pmatrix} U^\dagger + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right\}. \tag{15}$$

Then the oscillation probabilities are calculated, e.g., as

$$P(\nu_\mu \to \nu_e; E, L) = 4 \sin^2 \frac{\delta m_{31}^2 L}{4E} c_\phi^2 s_\phi^2 s_\psi^2 \left\{ 1 + \frac{a}{\delta m_{31}^2} \cdot 2(1 - 2s_\phi^2) \right\}$$

$$+ 2 \frac{\delta m_{31}^2 L}{2E} \sin \frac{\delta m_{31}^2 L}{2E} c_\phi^2 s_\phi s_\psi \left\{ -\frac{a}{\delta m_{31}^2} s_\phi s_\psi (1 - 2s_\phi^2) + \frac{\delta m_{21}^2}{\delta m_{31}^2} s_\omega (-s_\phi s_\psi s_\omega + c_\delta c_\psi c_\omega) \right\}$$

$$- 4 \frac{\delta m_{21}^2 L}{2E} \sin^2 \frac{\delta m_{31}^2 L}{4E} s_\delta c_\phi^2 s_\phi c_\psi s_\psi c_\omega s_\omega. \tag{16}$$

As stated, oscillation probabilities such as $P(\bar{\nu}_\mu \to \bar{\nu}_e)$, $P(\nu_e \to \nu_\mu)$ and $P(\bar{\nu}_e \to \bar{\nu}_\mu)$ are given from the above formula by some appropriate changes of the sign of a and/or δ.

The first condition (10) of the approximation leads to a constraint for the baseline length of long-baseline experiments as

$$L \ll 1.72 \times 10^3 \text{km} \left(\frac{\rho}{3\text{g cm}^{-3}} \right) \tag{17}$$

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The second condition (11) gives the energy region where we can use the approximation,

$$E \gg 76.0\text{MeV} \left(\frac{\delta m_{21}^2}{10^{-4}\text{eV}^2} \right) \left(\frac{L}{300\text{km}} \right). \tag{18}$$

As long as these conditions, (17) and (18) are satisfied, the approximation (12) works pretty well(15; 32).

2.2 Magnitude of CP violation and matter effect

The available neutrino as an initial beam is ν_μ and $\bar{\nu}_\mu$ in the current long baseline experiments(33; 34). The "CP violation" gives the nonzero difference of the oscillation probabilities between, e.g., $P(\nu_\mu \to \nu_e)$ and $P(\bar{\nu}_\mu \to \bar{\nu}_e)$(15). This gives

$$
\begin{aligned}
P(\nu_\mu \to \nu_e; L) - P(\bar{\nu}_\mu \to \bar{\nu}_e; L) = & \, 16 \frac{a}{\delta m_{31}^2} \sin^2 \frac{\delta m_{31}^2 L}{4E} c_\phi^2 s_\phi^2 s_\psi^2 (1 - 2s_\phi^2) \\
& - 4 \frac{aL}{2E} \sin \frac{\delta m_{31}^2 L}{2E} c_\phi^2 s_\phi^2 s_\psi^2 (1 - 2s_\phi^2) \\
& - 8 \frac{\delta m_{21}^2 L}{2E} \sin^2 \frac{\delta m_{31}^2 L}{4E} s_\delta c_\phi^2 s_\phi c_\psi s_\psi c_\omega s_\omega.
\end{aligned}
\tag{19}
$$

The difference of these two, however, also includes matter effect, or the fake CP violation, proportional to a. We must somehow distinguish these two to conclude the existence of CP violation as discussed in ref.(15).

On the other hand, a muon ring enables to extract ν_e and $\bar{\nu}_e$ beam. It enables direct measurement of pure CP violation through "T violation", e.g., $P(\nu_\mu \to \nu_e) - P(\nu_e \to \nu_\mu)$ as

$$P(\nu_\mu \to \nu_e) - P(\nu_e \to \nu_\mu) = -8 \frac{\delta m_{21}^2 L}{2E} \sin^2 \frac{\delta m_{31}^2 L}{4E} s_\delta c_\phi^2 s_\phi c_\psi s_\psi c_\omega s_\omega. \tag{20}$$

Note that this difference gives pure CP violation.

By measuring "CPT violation", e.g. the difference between $P(\nu_\mu \to \nu_e)$ and $P(\bar{\nu}_e \to \bar{\nu}_\mu)$, we can check the matter effect.

$$P(\nu_\mu \to \nu_e; L) - P(\bar{\nu}_e \to \bar{\nu}_\mu; L) = 16 \frac{a}{\delta m_{31}^2} \sin^2 \frac{\delta m_{31}^2 L}{4E} c_\phi^2 s_\phi^2 s_\psi^2 (1 - 2s_\phi^2)$$

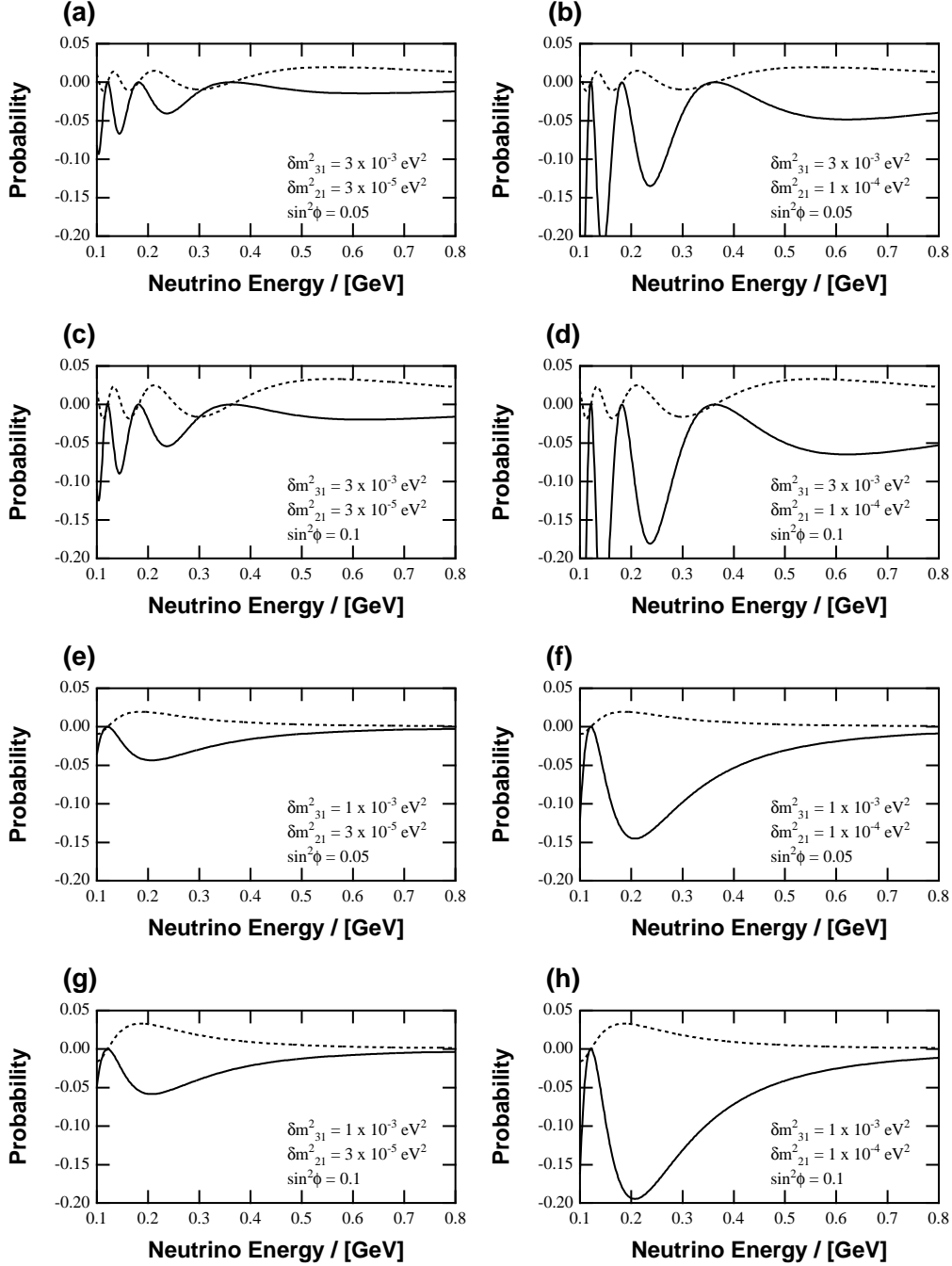Fig. 1. Graphs of $P(\nu_\mu \to \nu_e) - P(\nu_e \to \nu_\mu)$ (solid lines; pure CP-violation effects) and $P(\nu_\mu \to \nu_e) - P(\bar\nu_e \to \bar\nu_\mu)$ (dashed lines; matter effects) as functions of neutrino energy. Parameters not shown in the graphs are taken as follows. $\sin^2 \omega = 1/2, \sin^2 \psi = 1/2, \sin \delta = 1; \rho = \text{g/cm}^3$ and $L = 300\text{km}$.

$$-4\frac{aL}{2E}\sin\frac{\delta m_{31}^2 L}{2E}c_\phi^2 s_\phi^2 s_\psi^2(1-2s_\phi^2)$$

(21)

6

We present in Fig.1 "T-violation" part (20) and "CPT-violation" part (21) for some parameters allowed by the present experiments(35) [2] with $\sin^2 \omega = 1/2$, $\sin^2 \psi = 1/2$, $\sin \delta = 1$ fixed. The matter density is also fixed to the constant value $\rho = 2.5\text{g/cm}^3$(31). Other parameters are taken as $\delta m_{31}^2 = 3 \times 10^{-3}\text{eV}^2$ and $1 \times 10^{-3}\text{eV}^2$, $\delta m_{21}^2 = 1 \times 10^{-4}\text{eV}^2$ and $3 \times 10^{-5}\text{eV}^2$.

"T-violation" effect is proportional to $\delta m_{21}^2/\delta m_{31}^2$ and, for $\phi \ll 1$, also to $\sin \phi$ as seen in eq.(20) and Fig.1. Recalling that the energy of neutrino beam is of several hundreds MeV, we see in Fig.1 that the "T-violation" effect amounts to at least about 5%, hopefully 10~20%. This result gives hope to detect the pure leptonic CP violation directly with the neutrino oscillation experiments.

3   CP violation in long long baseline experiments

Here we consider neutrino experiments with baseline $L \sim 10000\text{km}$ and see that "T violation" will be amplified(37; 38).

Since the baseline length L does not satisfy the condition (17,) we cannot make use of the previous approximation.

However, as $a, \delta m_{21}^2 \ll \delta m_{31}^2$ is satisfied, we have approximation formulae of the mixing matrix in matter $\tilde{U}$ for *neutrino*,

$$\tilde{U} = e^{i\psi\lambda_7}\Gamma e^{i\phi\lambda_5}e^{i\tilde{\omega}\lambda_2} \tag{22}$$

$$\tan 2\tilde{\omega} = \frac{\delta m_{21}^2 s_{2\omega}}{-ac_\phi^2 + \delta m_{21}^2 c_{2\omega}}$$

and of "masses" in matter $\tilde{m}_i^2$ for*neutrino*

$$(\tilde{m}_1^2, \tilde{m}_2^2, \tilde{m}_3^2) = (\lambda_-, \lambda_+, \delta m_{31}^2 + ac_\phi^2) \tag{23}$$

$$\lambda_\pm = \frac{ac_\phi^2 + \delta m_{21}^2}{2} \pm \frac{1}{2}\sqrt{(-ac_\psi^2 + \delta m_{21}^2 c_{2\omega})^2 + (\delta m_{21}^2 s_{2\omega})^2}.$$

Thus the "T violation" is given by

$$P(\nu_e \to \nu_\mu; L) - P(\nu_\mu \to \nu_e; L)$$
$$= 4s_\delta c_\phi^2 s_\phi c_\psi s_\psi c_{\tilde{\omega}} s_{\tilde{\omega}} \left(\sin \frac{\delta\tilde{m}_{21}^2 L}{2E} + \sin \frac{\delta\tilde{m}_{32}^2 L}{2E} + \sin \frac{\delta\tilde{m}_{13}^2 L}{2E} \right) \tag{24}$$

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[2] Although the Chooz reactor experiment have almost excluded $\sin^2 \phi = 0.1$(36), there remains still small chance to take this value.

$$\sim s_\delta c_\phi^2 s_\phi c_\psi s_\psi c_{\tilde{\omega}} s_{\tilde{\omega}} \left(\sin \frac{\delta \tilde{m}_{21}^2 L}{2E} \right),$$

here in the last equation we dropped the terms $\sin \frac{\delta \tilde{m}_{32}^2 L}{2E} + \sin \frac{\delta \tilde{m}_{13}^2 L}{2E}$, since they oscillate very rapidly and will no contribution to the actual measurement.

As is seen in eq. (22), due to MSW effect(28; 29) "T violation" may be amplified very much even if the mixing angle ω is very small and hence we can test whether there is a CP phase δ(38).

4 Summary and conclusion

We considered how large CP/T violation effects can be observed making use of low-energy neutrino beam, inspired by PRISM(23).

First we consider the baseline with several hundreds km. In this case more than 10%, hopefully 20% of the pure CP-violation effects may be observed within the allowed region of present experiments. To see CP-violation effect those baseline length and the neutrino energy are most preferable statistically(32).

Then we consider the baseline with ~ 10000 km. We see that in this case, due to MSW effect, "T violation" will be amplified and we can test whether the CP phase δ is large or not.

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